Mathematics for Chemists **Easter Vacation 2005**
http://timmel.chem.ox.ac.uk/lectures/

Partial Differential Equations

1. Which of the following functions *u(x,y,t)* are solutions of the partial differential equation for the vibration of plates: $u_{tt} = u_{xx} + u_{yy}$?

(Notation: remember that $u_{tt} = \dfrac{\partial^2 u}{\partial t^2}$.)

a) $u = \sin(\sqrt{2}t)\sin x \sin y$
b) $u = C_1 t + C_2 x + C_3 y + C_4$ with C_i = constant
c) $u = e^{txy}$
d) $u = txy$

2. Solve the following partial differential equations for $u = u(x,y,z)$ using elementary methods (methods you are familiar with from ordinary differential equations are sufficient):

a) $u_{xy} + u_x = 2x + y$ (Hint: use $v(x,y) = u_x$ and solve for v first)
b) $xu_{xy} = u_y + y$ (Hint: use $v(x,y) = u_y$ and solve for v first)

3. Find particular solutions of the following partial differential equations using the product approach, *e.g.,* $u(x,t) = X(x)T(t)$:

a) $2u_t + 3u_x - 2u = 0, u = u(x,t)$
b) $v_t + (1+A)v_x = 0, A = const, v = v(x,t)$
c) $u_{xx} + 2u_{xy} + u_y = 0, u = u(x,y)$
d) $u_{xx} + u_{yy} + \lambda^2 u = 0, \lambda > 0; u = u(x,y)$

4. Solve equations 1-3 in the problem booklet (11.).

And just so you do not forget last term's efforts, some reminders

5. **Complex numbers**

a) Find the real and imaginary parts of the following complex numbers:

(i) $\ln(7i)$ (ii) $\sin(i)$ (iii) $(i+1)^{(i+1)}$

b) Draw the solutions to the following equation in an Argand Diagram

$$z^3 = (-\sqrt{3} + i)$$

c) Show that

$$1 + (\cos 2\theta + i \sin 2\theta)^6 = 2\cos 6\theta(\cos 6\theta + i \sin 6\theta) .$$

6. <u>Integration</u>

a) $\int \frac{2x^2 + 9x + 12}{x^2 + 6x + 10} dx$

b) $\int \cos x \cos 3x dx$

c) $\int (3x^2 + 4x) \ln(x+1) dx$

d) $\int \frac{1 + \cos x}{\sin^3 x} dx$

d) Show that

$$\int_0^{\pi/2} \ln(\sin x) dx = \int_0^{\pi/2} \ln(\cos x) dx = \frac{1}{2} \int_0^{\pi/2} \ln(\sin 2x) dx - \frac{\pi}{4} \ln 2$$

And hence evaluate

$$\int_0^{\pi/2} \ln(\sin x) dx .$$

7. <u>Differential Equations</u>

Solve the following differential equations:

a) $\cos x \frac{dy}{dx} = e^y$

b) $\frac{dy}{dx} + \frac{y}{x} = \sin^2 x$

c) $y'' + 4y' + 3y = x^2 e^x$

d) $\frac{d\ y}{dx} + 8\frac{dy}{dx} + 12y = xe^{-2x}$

e) $(x^2 + y^2)(x + yy') = y + xy'$

Hint: Examine if e) is an exact DE.